

## PRINCIPLE STRESS IN SHEAR CAPACITY,

Daniel Christianto<sup>1\*</sup>, Tavio<sup>2</sup>, Helga Lenita<sup>3</sup>, Rafael Dean Widjaja<sup>4</sup>, Amanda Greta  
Theodora<sup>5</sup>, Yenny Untari Liucius<sup>6</sup>

<sup>1</sup> Civil Engineering Department, Universitas Tarumanagara, Jakarta, Indonesia

Email: [danielc@ft.untar.ac.id](mailto:danielc@ft.untar.ac.id)

<sup>2</sup> Civil Engineering Department, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia\*

Email: [tavio@its.ac.id](mailto:tavio@its.ac.id)

<sup>3</sup> Civil Engineering Department, Universitas Tarumanagara, Jakarta, Indonesia

Email: [helga.325210047@stu.untar.ac.id](mailto:helga.325210047@stu.untar.ac.id)

<sup>4</sup> Civil Engineering Department, Universitas Tarumanagara, Jakarta, Indonesia

Email: [rafael.325210013@stu.untar.ac.id](mailto:rafael.325210013@stu.untar.ac.id)

<sup>5</sup> Civil Engineering Department, Universitas Tarumanagara, Jakarta, Indonesia

Email: [amanda.325210054@stu.untar.ac.id](mailto:amanda.325210054@stu.untar.ac.id)

<sup>6</sup> Civil Engineering Department, Universitas Tarumanagara, Jakarta, Indonesia

Email: [yenny@ft.untar.ac.id](mailto:yenny@ft.untar.ac.id)

\*Corresponding Author

Submitted: 15-03-2023, Revised: 17-03-2023, Accepted: 18-03-2023

---

### ABSTRACT

*The design and construction of reinforced concrete high-rise building structures are currently changing for the better, this is due to technological advances in the base materials of concrete that leads to a higher quality. A higher quality concrete in multi-story buildings can reduce the cross-sectional dimensions of lateral resisting elements (earthquake forces). Normal concrete is formed from aggregates (coarse + fine) and paste (a mixture of cement, sand, water, and additives) in general. High-quality concrete is achieved through the use of innovative materials such as steel fiber. Additionally, to form ultra-high concrete, it is made to be close to homogeneous. For this reason, the use of coarse aggregates is eliminated, hence fine aggregates can blend perfectly with cement and other innovative ingredients to form a concrete paste with higher strength than coarse aggregates. Concrete structural elements cannot be separated from shear problems. Shear forces generally combine with bending, torsion, and axial forces. Therefore, it is important to understand the interaction between shear and other forces.*

**Keywords:** Shear strength, principal stress, longitudinal reinforcement

## 1. INTRODUCTION

The design and construction of reinforced concrete high-rise building structures are constantly changing for the better. This is due to technological advances in the base materials of concrete that lead to higher quality and high-quality concrete in multi-story buildings that can reduce the cross-sectional dimensions of lateral resisting elements (earthquake forces). Normal concrete is formed from aggregates (coarse + fine) and paste (a mixture of cement, sand, water, and additives). In general, through the application of innovative materials such as steel fiber, high-quality concrete is achieved. Additionally, to form ultra-high concrete, it is made to be close to homogeneous. For this reason, the use of coarse aggregates is eliminated, hence fine aggregates can blend perfectly with cement and other innovative ingredients to form a concrete paste with higher strength than coarse aggregates.

The concrete structural element cannot be separated from shear problems. Shear forces generally combine with bending, torsion, and axial forces. Therefore, it is imperative to understand the interaction between shear and other surrounding forces. Published tests have shown that shear failure of concrete structural elements is brittle - occurring suddenly without warning because the shear strength/capacity of concrete structures depends on the tensile and compressive strength of the Concrete material (principal stresses). In addition, this is different when the goal is designing a ductile structure. Even though it is puzzling to predict shear failures, it is considerably advised to be averted at all costs. Shear failure on structural elements can cause a large disaster, property damage, and casualties. In the year 1955, two of the US Air Force bases collapsed due to their weight (Yang, 2014). All these were caused by shear stress because the roof's structure did not use a transversal reinforcement. In the year 2006, a part of the de la Concorde de Laval's flyover in Quebec Canada collapsed and caused the loss of life. Studies found that the main girder beam is designed following the guidelines that were formed forty years ago, and it also ignores the use of a transversal reinforcement.

The behavior and phenomena of shear in a concrete structure need to be brought to attention because it is highly affected by the forming materials. This research on shear has sparked many interesting discussions and becomes a staple debate topic among researchers. The formula for shear found in ACI 318 is for normal concrete, and Join ACI-ASCE committee 426 (1973) states that rough aggregates are a component that contributes to shear strength. Hence, researchers began their research on shear strength without rough aggregates and studied the failure pattern with principal stress.

## 2. RESEARCH METHOD

Published tests have shown that shear failure of concrete structural elements are brittle - occurring suddenly without warning. This is because the shear strength/capacity of concrete structures depends on the tensile and compressive strength of the concrete material (principal stresses). This research tested the structural elements of concrete beams without coarse aggregate using the principle of the third point loading test with variations in longitudinal reinforcement. The test results show that shear strength is related to the longitudinal reinforcement and has the same graphical shape as the principal stress.

A structural element, such as concrete and steel, will never be free from the issue of shear. Shear forces generally combine with bending, torsion, and axial forces. Therefore, it is important to understand the interaction between shear and other forces, especially the ones that involve the strength of reinforced concrete.

Published tests have shown that shear failure of concrete structural elements is brittle - occurring suddenly without warning. This condition is the opposite of the general goal in designing a structure, where a ductile structure is ideal so that failures can be detected as early as possible.

Shear behavior is different for each structure. In order to know the shear behavior of reinforced concrete, the understanding of shear behavior on structures that are homogenous, isotropic, and linearly elastic is needed. With modifications, an approach based on said assumptions can be applied to produce a viable image of the formation of cracks and shear strength of reinforced concrete.

### Shear Tension on Uncracked Elastic Beam

Shear force and shear tension appears in the part of a beam that is affected by a change in torque. With respect to the traditional theory of uncracked beam that is elastic and homogenous, shear tension ( $v$ ) in elements 1 and 2 of a beam (Figure 1.) is analyzed with the formula:

$$v = \frac{VQ}{lb} \quad (2.1)$$

- $v$  : shear force at the cross-section,
- $Q$  : a static moment of the cross-sectional area above where the stress is being observed,
- $I$  : a moment of inertia of the section,
- $b$  : the width of the cross-section where the stress is observed.

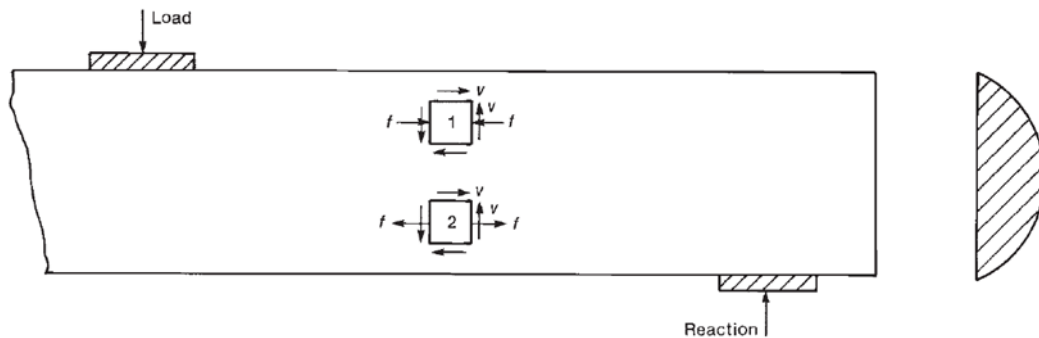


Figure 1. Shear Stress Distribution in a Homogeneous Uncracked Rectangular Beam

[2]

For an uncracked rectangular beam, Equation (2.1) yields the shear force distribution shown in Figure 1. When an element is subjected to normal stress and shear stress, there will be principal stresses. The principal stresses are the largest and smallest normal stresses working on the element that can be analyzed using Mohr's circle for stress. If element 2 below the neutral line in Figure 1. is subjected to tensile stress  $f_t$  and shear stress  $v$ , then the principal stress can be calculated by

$$\text{Principal tensile stress} \quad : \quad f_{t,max} = \frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + v^2} \quad (2.2)$$

$$\text{Principal compressive stress} \quad : \quad f_{c,max} = \frac{f_t}{2} - \sqrt{\left(\frac{f_t}{2}\right)^2 + v^2} \quad (2.3)$$

Principal stress direction : 
$$\tan 2\theta_{maks} = \frac{v}{f_t/2} \quad (2.4)$$

The behavior of non-homogeneous concrete, Prestressed concrete beams, and reinforced concrete is different from steel. The tensile strength of concrete is approximately only 1/10 of the compressive strength, so it's easy for cracks to occur due to tensile principal stresses. In element 1 above the neutral line, cracking will not occur because the maximum principal stress is compressive. At element 2 below the neutral line, the maximum principal stress is tensile, so cracking can occur. As you get closer to the support, the bending stress  $f$  decreases, while the shear stress  $v$  increases, so that the tensile principal stress acts at an angle of about  $45^\circ$  in the support area.

Since the tensile strength of concrete is very low, diagonal tensile cracking will occur in this support section. To prevent this type of cracking, special reinforcement called diagonal tensile reinforcement is required. The principal stress trajectories of a beam with uniform loading are shown in Figure 2. The solid line shows the tensile principal stress trajectory and the dashed line shows the compressive principal stress trajectory. From these principal stress trajectories, the direction of the cracks that will occur can be estimated.

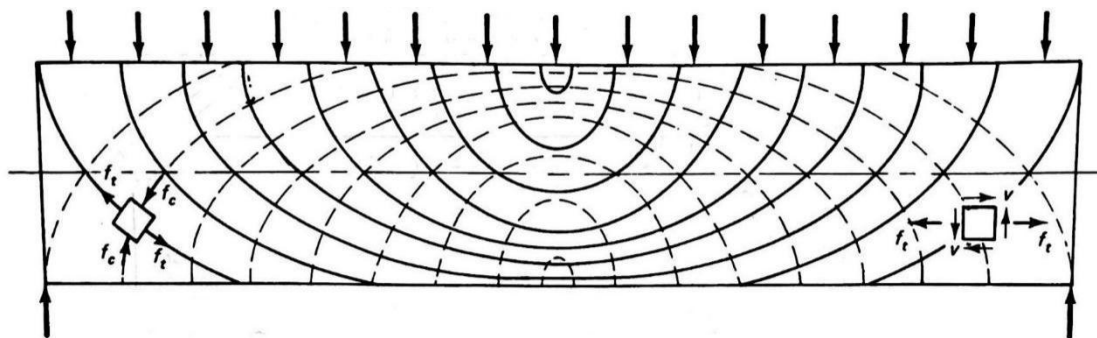


Figure 2. Principal Stress Trajectory of Homogeneous Isotropic Beam.

Solid Line: Pull Trajectory, Break Line: Push Trajectory

[3]

In structural components without transverse reinforcement, shear is assumed to be resisted by concrete. The shear strength provided by the concrete ( $V_c$ ) is taken as the shear that causes an inclination crack. After rationally determining which variables have an effect, the relationship between the variables affecting the shear strength of concrete is statistically determined from the test results.

The assumed shear strength is reached when the ultimate tensile stress ( $f_{t,max}$ ) is equal to the tensile stress of the concrete, which is considered to be proportional to  $\sqrt{f_c}$ . Also, assume that the flexural tensile stress ( $f_t$ ) is proportional to  $E_c/E_s$  times the tensile stress in the reinforcement and the shear stress ( $v$ ) is proportional to the average shear stress. Then, assume that the modulus of

elasticity of concrete ( $E_c$ ) is proportional to  $\sqrt{f_c'}$  and  $V_n$  and  $M_n$  is the nominal shear force and nominal bending moment in a cross-section when an inclination crack forms. Based on these assumptions, we obtain

$$v = k_1 \frac{V_n}{b_w d} \quad (2.5)$$

$$f_t = k_2 \frac{E_c}{E_s} f_s = k_3 \frac{E_c}{E_s} \frac{M_n}{d A_s} = k_4 \frac{\sqrt{f_c'}}{E_s} \frac{M_n}{d A_s} = \frac{k_4}{E_s} \frac{M_n \sqrt{f_c'}}{\rho_w b_w d^2} \quad (2.6)$$

$$f_{t,max} = k_5 \sqrt{f_c'} \quad (2.7)$$

Where  $k_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ , and  $k_5$  are dimensionless coefficients (unitless) and the value of the elastic modulus of steel reinforcement ( $E_s$ ) is known. In Section 2.2.1, the ultimate tensile stress is analyzed by equation (2.2) and rewritten as follows:

$$f_{t,max} = \frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + v^2} \quad (2.2)$$

By substituting Equations (2.5) to (2.7) into Equation (2.2), obtained

$$k_5 \sqrt{f_c'} = \frac{V_n}{b_w d} \left( \frac{1}{2} \times \frac{k_4}{E_s} \times \frac{M_n \sqrt{f_c'}}{\rho_w V_n d} + \sqrt{\left(\frac{1}{2} \times \frac{k_4}{E_s} \times \frac{M_n \sqrt{f_c'}}{\rho_w V_n d}\right)^2 + k_1^2} \right) \quad (2.8)$$

$$\frac{V_n}{b_w d \sqrt{f'_c}} = k_5 \left( \frac{1}{2} \times \frac{k_4}{E_s} \times \frac{M_n \sqrt{f'_c}}{\rho_w V_n d} + \sqrt{\left( \frac{1}{2} \times \frac{k_4}{E_s} \times \frac{M_n \sqrt{f'_c}}{\rho_w V_n d} \right)^2 + k_1^2} \right)^{-1} \quad (2.9)$$

by inputting data on the modulus of elasticity of materials ( $E_s$  dan  $E_c$ ) in equation (2.9), a graph will be obtained as shown in Figure 3.

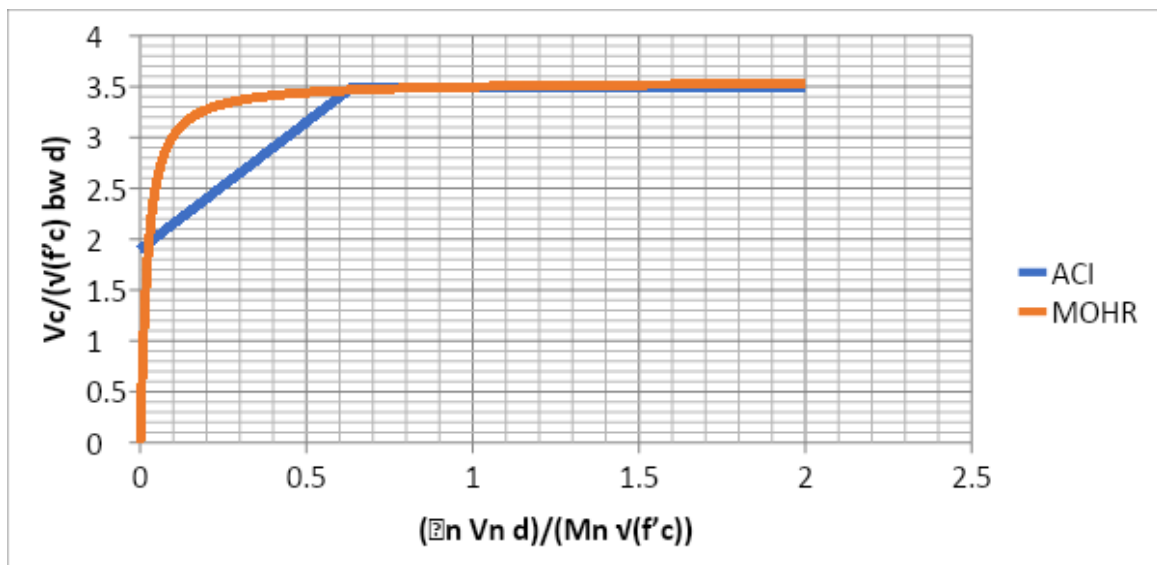


Figure 3.

Source: Principal Stresses Based on Mohr's Circle

### 3. RESULTS AND DISCUSSIONS

#### Analysis of Longitudinal Reinforcement Effect

In this research, 70 mm × 125 mm × 1100 mm beams were used with longitudinal reinforcement area ratios varying from the minimum value to the maximum value according to Table 1. Shear arm ratio  $a/d$  in this test was 45/10.5. The concrete quality ( $f_c'$ ) of the beams was planned to be 80 MPa. The size and modeling of the beam specimens can be seen in Figure 4.

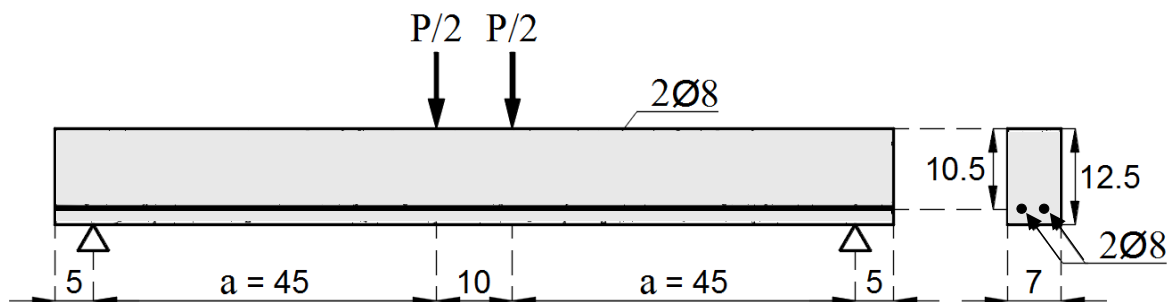


Figure 4.

Source: Example of 2Ø8 Reinforcement Beam Modeling

Table 1.

Source: Variation of Longitudinal Reinforcement Area

Reinforced	$\rho$
2Ø6	0.00769
2Ø8	0.01367
2Ø10	0.02136
2Ø12	0.03076
2Ø16	0.05468
2Ø19	0.07711

### Concrete Compressive and Shear Strength Test Result

The result of the cylinder compressive strength and beam shear strength tests can be seen in Table 2.

Table 2.

Source: Test Result of Cylinder Compressive Strength and Beam Shear Force

Ø (mm)	Code	Silinder (Ø10 cm × 20 cm)		Beam (7 × 12,5 × 110 cm <sup>3</sup> )		Curing time (day)
		Force (kN)	f <sub>c</sub> (MPa)	Force (kN)	Type of failure	
6	A21	395.1	50.3	10.31	Bend	58
6	A22	516.7	65.72	10.13	Bend	58
8	A11	609.1	77.52	14.22	Bend + Shear	58
8	A12	583.3	74.23	18.55	Bend + Shear	58
10	A31	845.4	107.6	24.31	Bend + Shear	58
10	A32	867.3	110.4	25.52	Bend + Shear	58
12	A41	559.4	71.20	20.62	Shear	58
12	A42	489.9	62.35	23.49	Shear	58
16	A51	624.7	79.51	34.03	Shear	58
16	A52	443.7	56.41	23.43	Shear	58
19	A61	602.5	76.68	29.59	Shear	58
19	A62	594.8	75.66	27.67	Shear	58

Based on the data in Table 2. it can be analyzed to obtain the ultimate shear force  $V_u$ , concrete shear capacity  $V_c$ , nominal moment  $M_n$ , ultimate moment  $M_u$ , which are shown in Table 3.

Table 3.

Source: Recapitulation of Shear Strength Analysis on Beam Test Objects

Ø	f <sub>c</sub> average (MPa)	P average (kN)	V <sub>c</sub> (kN)	M <sub>n</sub> (kNm)	P <sub>m</sub> (kN)	P <sub>v</sub> (kN)	Failure
6	58	10.22	9.3329	2.0366	8.9361	18.4558	Bend
8	75.9	16.38	10.6733	3.5838	15.8126	21.1366	Bend + Shear
10	109	24.92	12.7894	5.5805	24.6869	25.3687	Bend + Shear
12	66.8	22.06	10.0121	7.5761	33.5559	19.8142	Shear
16	68	28.73	10.1027	12.4067	55.0254	19.9954	Shear
19	76.1	28.63	10.6849	16.5818	73.5815	21.1598	Shear



From the test results above, the type of failure that happens in concrete beams with a reinforcement diameter of 8 mm and 10 mm are bending + shear. The beam that is tested suffered from bend failure first, then a crack from shear, and finally causing a shear failure. The graph of the longitudinal reinforcement ratio effect on the shear capacity of a normal concrete beam without transversal reinforcement can be seen in Figure 5.

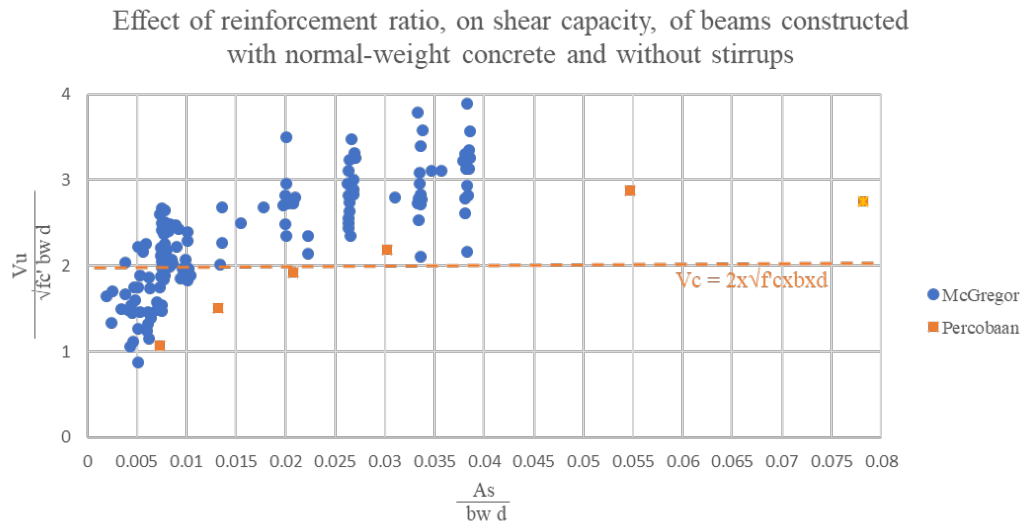


Figure 5.

Source: Effect of Longitudinal Reinforcement Ratio towards Concrete Shear Capacity

The relationship of Longitudinal Reinforcement Ratio and  $V_u/V_c$  is plotted to show the effect of longitudinal reinforcement towards  $V_u/V_c$  as shown in Figure 6.

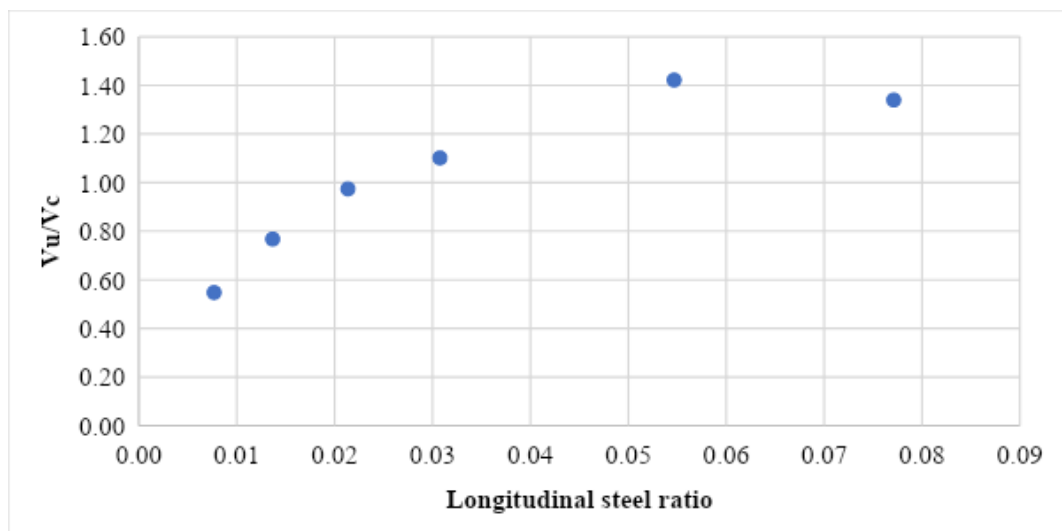


Figure 6.

Source: Relationship of Longitudinal Reinforcement Ratio and  $V_u/V_c$

#### 4. CONCLUSIONS AND SUGGESTIONS

Based on the discussion of the test results above, the following conclusions can be drawn:

1. For the condition of minimum longitudinal reinforcement ratio, transversal (AV) is also needed.
2. The area of longitudinal reinforcement contributes largely to shear strength.
3. This is shown with the shear strength ratio constantly under 2 when the longitudinal reinforcement is minimum and proportionally increases with the increase in longitudinal reinforcement ratio.
4. The graphical shape of the test is very similar to the graph of principal stress.

#### ACKNOWLEDGEMENT

The authors would like to gratefully acknowledge those who have contributed and supported to make this research possible, particularly for the technical support and assistance during the experimental work in the Laboratory of Construction Material Technology, Department of Civil Engineering, Tarumanagara University, Jakarta.

#### REFERENCES (APA 7<sup>TH</sup> EDITION)

##### Electronic Journals:

[2] *Reinforced concrete: mechanics and design* (8th ed.).

[3] *Reinforced concrete: A fundamental approach*.

Kani GNJ. (1964). The riddle of shear failure and its solution. *ACI J Proc*, 61(4):411-468. <https://doi.org/10.14359/7791>

Kani GNJ. (1966). Basic facts concerning shear failure. *ACI J Proc*, 63(6):675-692. <https://doi.org/10.14359/7644>

Kani GNJ. (1967). How safe are our large reinforced concrete beams? *ACI J Proc*, 64(3):128-141. <https://doi.org/10.14359/7549>

Kani GNJ. (1969). A rational theory for the function of web reinforcement. *ACI J Proc*, 66(3):185-197. <https://doi.org/10.14359/7354>

Leonhardt. F., & Walther. R. (1965). Contribution to the treatment of shear in reinforced concrete. *National Research Council of Canada*, 1-183. <https://doi.org/10.4224/20386736>

Joint ASCE-ACI Committee 426. (1973). The shear strength of reinforced concrete members. *J Structural Division*. 99(6):1091-1187. <https://doi.org/10.1061/JSDEAG0003532>

Joint ACI-ASCE Committee 326 (1962). Shear and diagonal tension. *ACI J Proc*. 59(3):352-396.

ACI Committee 318. Building code requirements for structural concrete (ACI 318-19), 2019. <https://doi.org/10.14359/51716937>

- Sandjaya, A., Tavio, & Christianto, D. (2019). Experimental study of mortar compressive strength with Anadara Granosa powder as a substitute for partial use of cement,. *IOP Conference Series: Materials Science and Engineering*, 650(1):1-6. <https://doi.org/10.1088/1757-899X/650/1/012037>
- Kushartomo, W., Wiyanto, H., & Christianto, D. (2022). Effect of cement–water ratio on the mechanical properties of reactive powder concrete with marble powder as constituent materials. *Lecture Notes in Civil Engineering*, 216:177-185. [https://doi.org/10.1007/978-981-16-7949-0\\_16](https://doi.org/10.1007/978-981-16-7949-0_16)
- Christianto, D., Makarim, C. A., Tavio, & Liucius, Y. U. (2020). Size effect on shear stress of concrete beam without coarse aggregate. *Journal of Physics: Conference Series*, 1477(5): 1-7. <https://doi.org/10.1088/1742-6596/1477/5/052043>
- Christianto, D. Makarim, C. A., & Tavio. (2021). Influence of longitudinal reinforcement ratio on shear capacity of no coarse-aggregate concrete. *International Journal of GEOMATE*, 21(86):122-130. <https://doi.org/10.21660/2021.86.i2288>
- Christianto, D., Makarim, C. A., & Tavio. (2021). Effect of longitudinal steel reinforcement on shear capacity of SFRC beams without coarse aggregate. *TRKU*, 63(1):6909-6917.
- Christianto, D., Makarim, C. A., Tavio, T., & Pratama, I. D. (2022). Modified EC2's shear strength equation for no coarse aggregate RC beams. *International Journal on Advanced Science, Engineering and Information Technology*, 12(6):2211-2216. <https://doi.org/10.18517/ijaseit.12.6.15247>
- Christianto, D., Makarim, C. A., Tavio, & Jusuf, A. H. (2022) A proposed formula for predicting size effect on shear strength of concrete beams without coarse aggregate. *International Journal on Engineering Applications (IREA)*, 10(3):220-226. <https://doi.org/10.15866/irea.v10i3.20552>
- Christianto, D., Tavio, & Kurniadi, D. (2019). Effect of steel fiber on the shear strength of reactive powder concrete. *IOP Conference Series: Materials Science and Engineering* 508(1):1-10. <https://doi.org/10.1088/1757-899X/508/1/012006>
- Widjaja, W., Christianto, D., Kurniadi, D., Monica, L., Yuliana., & Michelle. (2019) Shear capacity of fiber-reinforced concrete beams without transverse reinforcement. *IOP Conf Ser Mater Sci Eng*, 650(1):1-12. <https://doi.org/10.1088/1757-899X/650/1/012036>
- Wulandari, M., Sofianto, M. F., & Tavio. (2020). Split tensile and flexural strength of concrete with artificial lightweight aggregate (ALWA) and steel-fiber. *Journal of Physics: Conference Series*, 1569(5): 1-7. <https://doi.org/10.1088/1742-6596/1569/4/042025>
- Sabariman, B., Soehardjono, A., Wisnumurti, Wibowo, A., & Tavio. (2020). Stress-strain model for confined fiber-reinforced concrete under axial compression. *Archives of Civil Engineerin*, 66(2):119-133. <https://doi.org/10.24425/ace.2020.131800>
- Kuchma, D. A., Wei, S., Sanders, D. H., Belarbi, A., & Novak, L. C. (2019). Development of one-way shear design provisions of ACI 318-19 for reinforced concrete. *ACI Structural Journal*, 116(4):285-295. <https://doi.org/10.14359/51716739>
- Rizqyani, N., Tavio. (2022) Proposed formula and comparison of shear strengths of reinforced concrete beams under various codes. *International Journal of GEOMATE*, 23(99):9-16. <https://doi.org/10.21660/2022.99.3497>

Pourbaba, M., Joghataie, A. (2019). Determining shear capacity of ultra-high performance concrete beams by experiments and comparison with codes. *Scientia Iranica*, 26(1):273-282. <https://doi.org/10.24200/sci.2017.4264>

**Books:**

Pincheira, J. A., Parra-Montesinos, G. J., Wang, C. K., & Salmon, C.G. (2021). *Reinforced concrete design* (9th ed.).