

Shear Strength Analysis of Reinforced Concrete Without Transverse Reinforcement using Finite Element Method

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Abstract. The efforts to obtain a higher quality concrete nowadays leads to the usage of more innovative materials, one of which is by removing coarse aggregate from the concrete mixture. This study was conducted to discuss the analysis of concrete beams without transverse reinforcement and without coarse aggregate using the finite element method which hasn't been done much. The research was done by modelling the concrete beam without coarse aggregate and shear reinforcement, and then the result is compared with the laboratory experiment done by Christianto, Tavio & Makarim (2020). Based on the study, we can conclude that the finite element method shows good agreements with the experimental results. Furthermore, both method shows the increasing in shear capacity with the increase of longitudinal reinforcement, however, when the reinforcement ratio exceeds 0,0547 (2Ø16) the shear capacity decrease. Moreover, both the finite element method and laboratory experiment results in shear capacity larger than the theoretic capacity based on ACI 318-14, which means that the ACI 318-14 formula is conservative enough to be used in design.

INTRODUCTION

The design of reinforced concrete building structures in the present has undergone so many changes to a better result, this is because the quality of the concrete has led to ultra-high concrete. Thus, the dimensions of the section size of reinforced concrete structures can be designed smaller than before. Efforts to achieve high quality concrete through the use of innovative materials such as powder concrete. [1] The development of powder concrete is based on the thought that on high strength concrete the coarse aggregates are the weakest element.

The analysis using the finite element method is one of the most effective methods to investigate the non-linear behavior of reinforced concrete structure. Study using this method can be done with lower cost compared to physical testing at the laboratory. [2]

There haven't been many research done to study about high strength concrete using the finite element method. Based on that reason the research about the effect of longitudinal reinforcement on the shear strength of concrete using the finite element method was done. The main purpose of this research is to gain better understanding on how beam without transverse reinforcement

Finite Element Method

The finite element method is a numerical method used to solve engineering and mathematic problems. Some of those problems include structural analysis, heat transfer, fluid current, mass transport and electromagnetic potential. [3]

Finite element involves structural modeling using interconnected tiny elements thus called the finite element. A function of displacement associated with each finite element. Each element connected directly or indirectly with other elements using nodal or interface element. Using the right stress/strain properties of the material used to model the structure, one can define the behavior of each node and element within the structure. The total package of equation depicting the behavior of each node better defined using matrices. [3]

Non-Linear Finite Element Method

Finite element method is one of valuable approach in analysing non-linear structure behaviours. There are several methods to solve the stiffness equations of non linear finite element. In structural mechanics, linear problem is when the stiffness matrices can be calculated only using the geometry and material property. In the non-linear case is when the stiffness matrices varied based on the load applied, and the load vector is based on the displacement. Non-linear condition in structural mechanics usually divided into 2 classes, geometric non linearity and material non linearity, which both affecting the structure deformation. Geometric non linearity can be seen from the changes in geometric configuration (such as deflection or buckling) and material non linearity can be seen from the change of material properties (such as plasticity). In the heat transfer, non-linear condition can be increased due to the temperature, depends on the conductivity or radiation, where the stiffness matrix is a function of temperature. [4]

When the load (P) works, the things to be calculated are displacement (u). Iterative procedure used to calculate for u. To find the load (P) several methods can be used, as follows [5]:

- Direct method
- Incremental method
- Newton-Ramphson method
- Modified Newton-Ramphson method
- Arc-length method

Interface Element

Interface element is element used to model cracking of material or relative movement from one section of the model. For examples interface element can be used to model concrete cracking, bond-slip relationship between concrete and steel reinforcement, and the mortar failure of masonry wall. [6]

Interface element is defined using the general formulation of finite element, where the thickness of this element is assumed to be zero. Definition of the relationship of traction and relative displacement on 3D plane can be seen at Fig. 1. For the purpose of defining an interface element with zero thickness based on the numerical analysis view point, a value of penalty stiffness needs to be assigned to the interface element. If the penalty stiffness that is assigned is too large it can cause numerical error, and if the value is too small it can result in unwanted relative displacement at both side of the interface element.

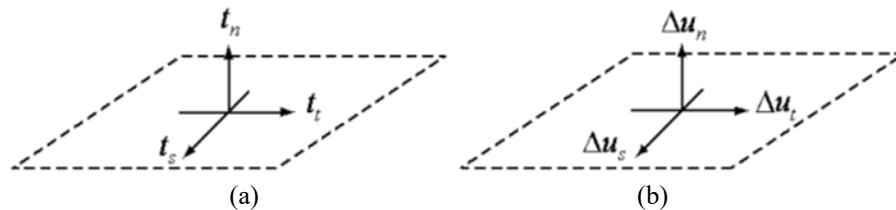


FIGURE 1. (a) Relative displacement and (b) traction on 3D plane [6]

Bond-Slip Interface

On reinforced concrete structures, the interaction of the steel reinforcement and the concrete is very complex, this interaction is defined by the longitudinal and secondary transverse cracking along the steel reinforcement. This behaviour can be modelled using the bond-slip mechanism where the slip relative to the steel reinforcement and the concrete explained phenomenologically. The mechanical behaviour of the slip zone is described using interface element with zero thickness.

Bond-slip behaviour that's been proposed mostly based the total deformation theory, which expressed traction as a function of total relative displacement. At MIDAS, the relationship of normal traction and normal relative displacement is assumed to be elastic linear, but the shear traction and slip is assumed as a non-linear function.

$$t_n = K_n \Delta u_n \quad (1)$$

$$t_t = f_t(dt) \quad (2)$$

Where t_n = normal traction, K_n = normal stiffness, Δu_n = normal displacement, t_t = shear traction, f_t shear displacement function, dt = slip.

Bond-slip Relationship of High Strength Concrete (FIB bulletin 10) [7]

Based on the Tests done, it is known that the maximum lateral bond stress increases as the compression strength increase. Fig. 2 show the relationship of bond strength ($\tau_{b, max}$) and compression strength (f_{cm}) of several pull out tests (high strength concrete and normal concrete). Based on those tests, Huang, Engstrom, & Magnusson [8] propose a relationship of compression strength and bond strength.

$$\tau_{b, max} = 0,45 f_{cm} \quad (3)$$

This equation is valid based on another test done using the same technique and type of reinforcement done by Lestander [9] and Hansend & Thorenfeldt [10] on Fig. 3.

The results disagree with the result of normal concrete. We know that the bond strength increases as the tensile strength of the concrete increase, proportional to the root or (power 2/3 of) the compression strength. This equation is formulated by Eigenhausen et al [11] for normal concrete.

$$\tau_{b, max} = 13,5 \left(\frac{f_{cm}}{30} \right)^\beta \quad (4)$$

Equation with $\beta=1/2$ had been adopted in CEB-FIP Model code 1990 [12] for maximum bond strength for “Good” bond condition. In the case of “Other” bond condition the same equation results in the bond strength reduced by 50%.

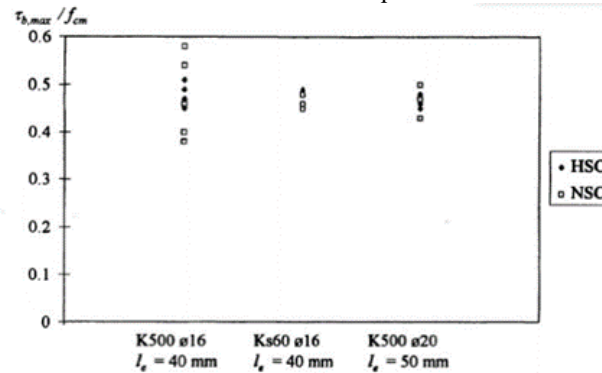


FIGURE 2. Relative bond strength $\tau_{b, max}/f_{cm}$ vs reinforcement type on pull out test with short embedment for high strength concrete (HSC) and Normal Strength Concrete (NSC) [13]

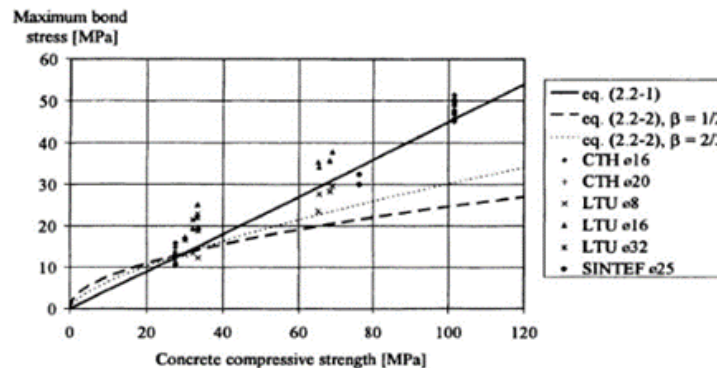


FIGURE 3. Comparison of several tests with equation 3 and equation 4, with $\beta= 1/2$ or $2/3$. Test results: CTH [13], LTU [9], SINTEF [10]. [7]

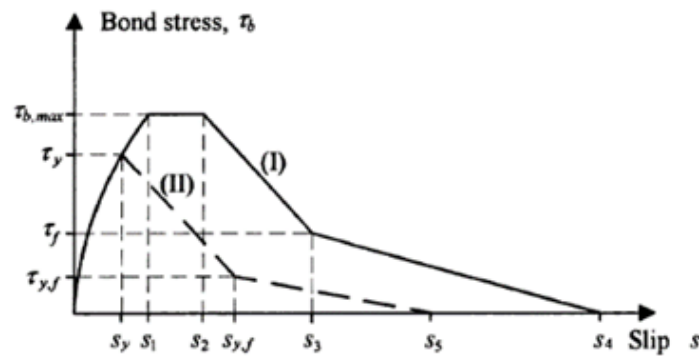


FIGURE 4. Relationship of bond stress and slip of high strength concrete [7]

In FIB bulletin 10, the bond-slip relationship of high strength concrete can be viewed on Fig. 4. Where the equation of the ascending curve is as follow

$$\tau_b = \tau_{b,max} \left(\frac{s}{s_1} \right)^\alpha \text{ untuk } s \leq s_1 \quad (5)$$

For other parameters of the curve can be seen at table 1 below.

TABLE 1. Simplified bond slip relationship for high strength concrete [7]

Parameter	Bond Condition	
	Good	Other
s_1	0.5 mm	0.5 mm
s_2	1.5 mm	1.5 mm
s_3	Clear rib spacing	Clear rib spacing
s_4	3 (rib spacing)	3 (rib spacing)
α	0.3	0.3
$\tau_{b,max}$	$0.45 f_{cm}$	$0.225 f_{cm}$
τ_f	$0.40 \tau_{b,max}$	$0.40 \tau_{b,max}$
$s_{y,f}$	$s_y + 0.25 \text{ mm}$	$s_y + 0.25 \text{ mm}$
s_5	2 (rib spacing)	2 (rib spacing)
$\tau_{y,f}$	$0.20 \tau_{b,max}$	$0.20 \tau_{b,max}$

RESEARCH METHOD

Research Samples

- This research done using a numerical method (finite element method). The primary data used in this research is a set of several laboratory testing done by Christianto, Tavio, & Makarim [14]. The testing data can be viewed on table 2, and the model of the research samples can be viewed on Fig. 5. These data used in the analysis to obtain the ultimate shear strength and the crack pattern of the beams.

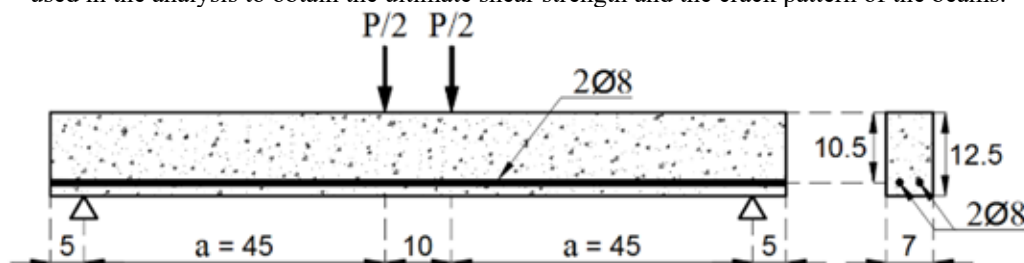


FIGURE 5. Concrete Beam Model [14]

TABLE 2. Sample data [14]

Kode Balok	Tulangan	ρ	f'_c (MPa)	P (kN)
E11	2ø6	0.73%	78.57	15.12
E12	2ø6	0.73%	81.04	14.73
E21	2ø8	1.32%	65.54	16.06
E22	2ø8	1.32%	61.02	17.37
E31	2ø10	2.08%	54.73	17.31
E32	2ø10	2.08%	69.25	23.13
E41	2ø12	3.02%	72.61	21.05
E42	2ø12	3.08%	58.18	23.81
E51	2ø16	3.47%	72.68	29.73
E52	2ø16	5.47%	74.40	29.94
E61	2ø19	7.82%	78.57	29.11
E62	2ø19	7.82%	70.65	27.70

Bond-Slip Parameters

The equation portrayed the relationship between the slip and traction of steel reinforcement and concrete calculated based on FIB bulletin 10 [7] for high strength concrete. The parameter used in the calculation can be viewed on table 3. For example, the relationship curve for sample E11F is given below on Fig. 6.

TABLE 3. Bond slip calculation parameter

Kode Balok	s1 (mm)	s2 (mm)	s3 (mm)	s4 (mm)	α	$\tau_{b,max}$	τ_f
E11F	0.5	1.5	4.2	12.6	0.3	35.36	14.14
E12F	0.5	1.5	4.2	12.6	0.3	36.47	14.59
E21F	0.5	1.5	6.3	18.9	0.3	29.49	11.80
E22F	0.5	1.5	6.3	18.9	0.3	27.46	10.98
E31F	0.5	1.5	7.0	21.0	0.3	24.63	9.85
E32F	0.5	1.5	7.0	21.0	0.3	31.16	12.47
E41F	0.5	1.5	10.2	30.6	0.3	32.67	13.07
E42F	0.5	1.5	10.2	30.6	0.3	26.18	10.47
E51F	0.5	1.5	12.6	37.8	0.3	32.71	13.08
E52F	0.5	1.5	12.6	37.8	0.3	33.48	13.39
E61F	0.5	1.5	13.3	39.9	0.3	35.36	14.14
E62F	0.5	1.5	13.3	39.9	0.3	31.79	12.72

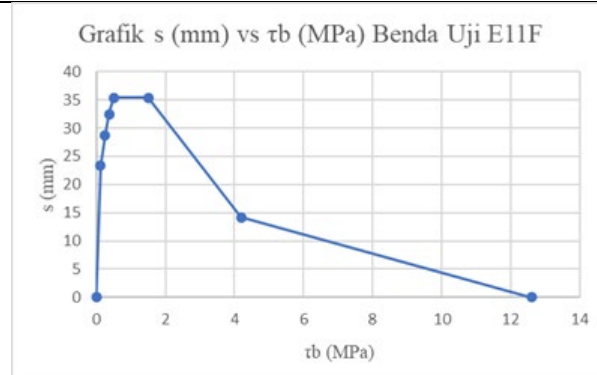


FIGURE 6. Bond slip curve of sample E11F

Sample Modelling

Structural Geometry

The beam modelled using box shaped solid element. The steel reinforcement modelled using cylinder shaped solid element. The steel reinforcement modelled using solid element rather than using line element, is so that the bond of the steel reinforcement surface and the concrete can be explicitly modelled using interface element (Fig. 7).

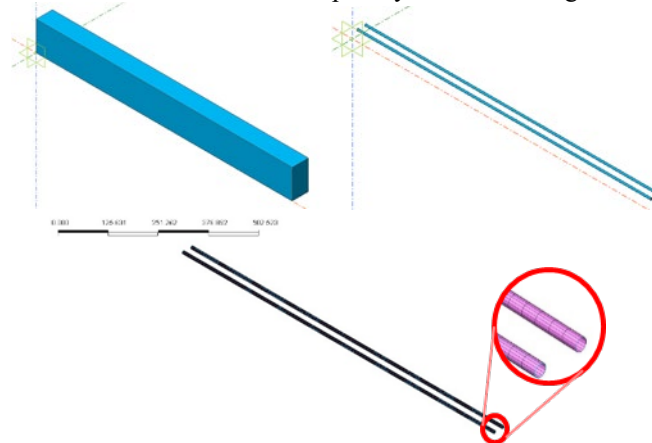


FIGURE 7. Geometry model of concrete beam, steel reinforcement and interface element

Materials

Each structural geometry is assigned with different materials and functions. For concrete elements, it is modelled using total strain crack option (Fig. 8), where the compression relationship modelled using thorenfeldt function (Fig. 9) and the tension relationship modelled using brittle function (Fig. 10). For steel elements, it is modelled using von mises option, by inputting the yield stress of the steel.

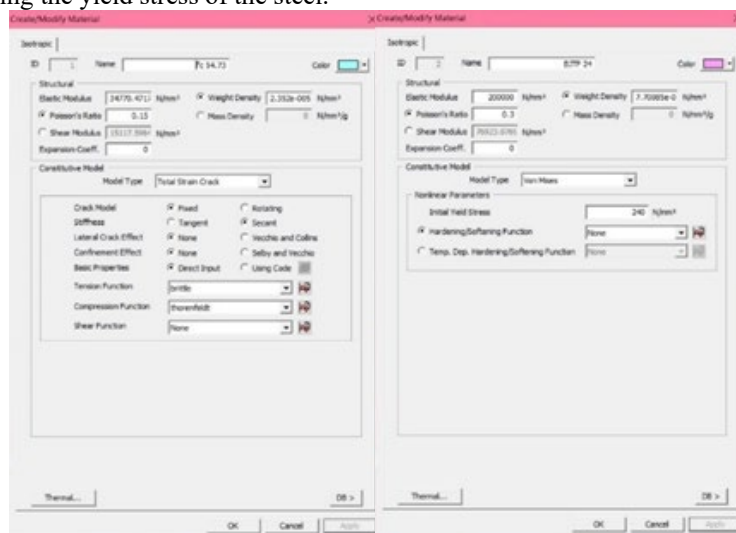


FIGURE 8. Material properties input

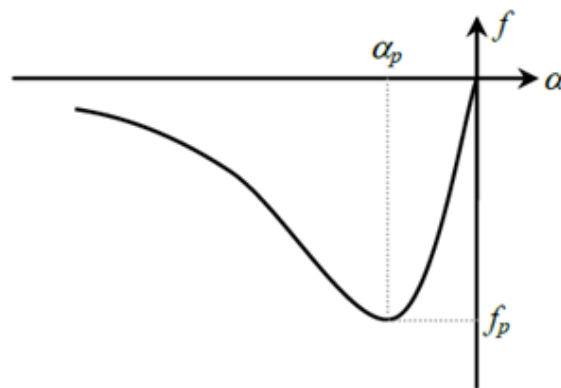


FIGURE 9. Stress-strain relationship based on Thorenfeldt function [6]

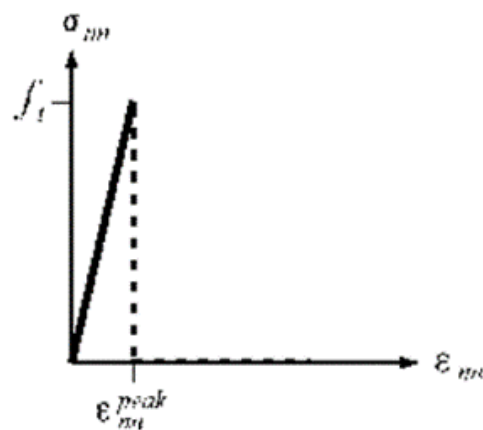


FIGURE 10. Stress-strain relationship based on brittle function [6]

Bond-slip parameters input

The bond of concrete and steel reinforcement is modeled using 2D interface element using the bond-slip non-linearity option and is modeled between the surface of the steel reinforcement and concrete. The interface element function is inputted using the multi-linear function, every ordinate of the slip and traction based on FIB bulletin 10 for high-strength concrete is inputted manually on the table provided (Fig. 11).

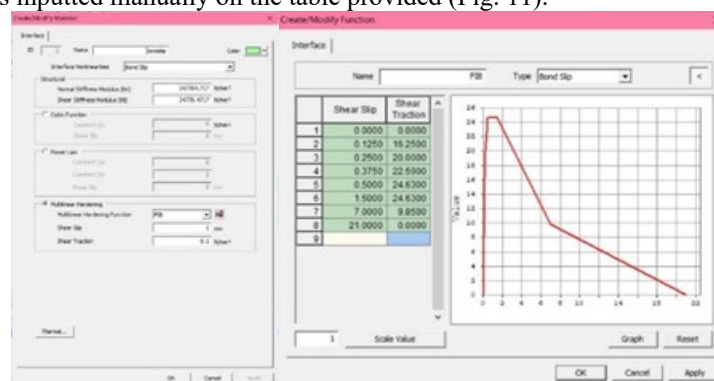


FIGURE 11. Bond slip parameter input

RESULTS AND DISCUSSION

Crack Pattern

Based on the analysis done using finite element method for each sample, it is obtained that the crack pattern shows similar result with those of the laboratory tests. The crack pattern when the sample failed is important to define the failure mode. The comparison of the crack patterns is shown below (Fig. 12 & 13).

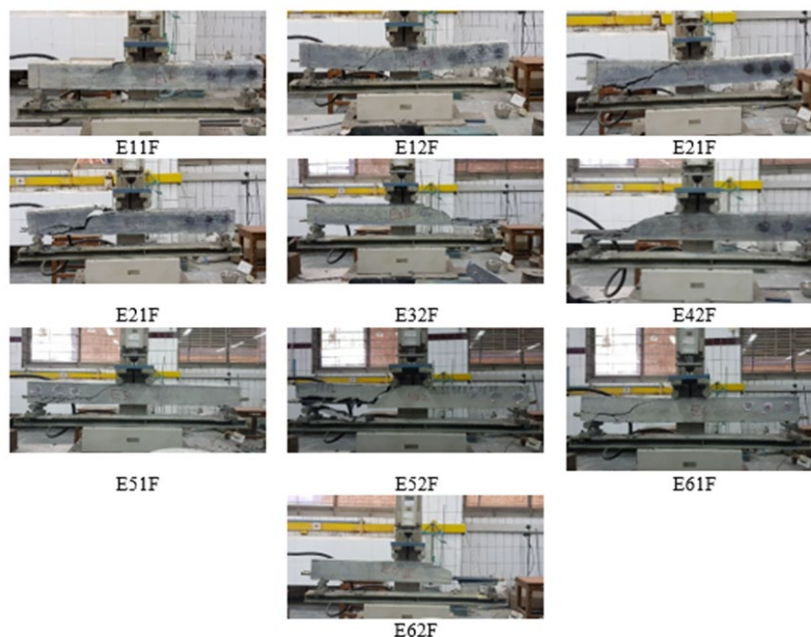


FIGURE 12. Crack pattern of laboratory test samples



FIGURE 13. Crack pattern of the finite element samples

Shear Capacity

Besides crack pattern, this research also aims to study the comparison of shear capacity based in finite element analysis and laboratory testing. The comparison of the force when the beam reach failure is shown on table 4 below.

TABLE 4. Comparison of force acting at the sample at failure of the laboratory test and finite element samples

No.	Code Balok	P Uji (kN)	P FEM (kN)
1	E11F	15.12	10.12
2	E12F	14.73	10.32
3	E21F	16.06	15.13
4	E22F	17.37	15.00
5	E31F	17.31	17.72
6	E32F	23.13	18.43
7	E41F	21.05	25.14
8	E42F	23.81	23.96
9	E51F	29.73	29.65
10	E52F	29.94	29.68
11	E61F	29.11	30.77
12	E62F	27.70	27.43

Based on above table, it can be seen that the finite element method and the laboratory testing show similar result. So that we can conclude that the finite modelling has been done well.

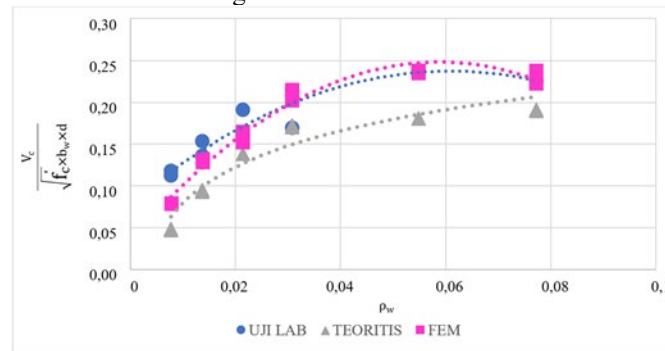


FIGURE 14. Reinforcement ratio (ρ_w) vs $\frac{V_c}{\sqrt{f'_c \times b_w \times d}}$

The graph on Fig. 14 above shows the comparison of the reinforcement ratio (ρ_w) with $\frac{V_c}{\sqrt{f'_c \times b_w \times d}}$, where V_c is the ultimate shear strength, f'_c is the compression strength, b_w width of the beam and d is the effective height of the beam. In this graph we can see that the result of finite element (magenta line) and laboratory testing (blue line) coincide with each other, this shows that the finite element result shows good agreement with laboratory testing. The shear capacity will increase along with the increase of the steel reinforcement used, and then when the reinforcement ratio reached 0.0308 the increment flattens and show a reduction when the reinforcement ratio exceeds 0.0547.

Besides that, based on the theoretic value (grey line), the finite element and test result show a larger value of shear strength, this shows that the theoretic equation used (ACI 318-14) is conservative enough to be used for design purposes.

CONCLUSION

Based on the research and analysis done, we can conclude that

1. The capacity and crack pattern of the finite element analysis show similar results with that of laboratory testing
2. Based on the finite element analysis and laboratory testing, it is shown that with the increase of steel longitudinal reinforcement, the shear capacity of the beam increase.

3. When the reinforcement ratio exceeds 0.0547, the shear capacity of the beam decrease
4. The shear strength based on finite element result and laboratory testing for every reinforcement ratio show larger value than its theoretical strength, this shows that the theoretic equation is conservative.

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